Special Relativity and Accelerated Motion

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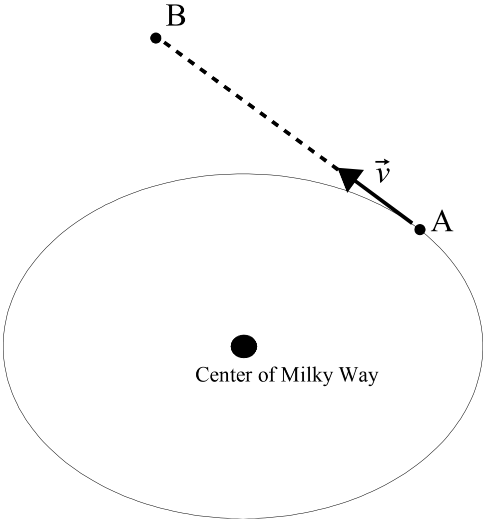
In this paper will be shown that acceleration does not affect in any way time dilation or length contraction effects expected by Lorentz transformations.

First, as an introduction, let us see what an inertial system is. By definition, an inertial system is one in which bodies move uniformly, with constant velocity.

A body can retain constant velocity (constant speed and unchanged direction) only for as long as there are no forces applied on it. For uniform motion we write:

$$\vec{v} = const.$$
$$a = 0$$

This is a theoretical possibility. However, physics does not know if a body can be freed from influence of other bodies, at any place in universe. To illustrate this, let us examine the case of a body which is isolated by many light years of vacuum from any other object: away from stars, planets, even cosmic dust. A good place that falls under that description would be somewhere on the periphery of our galaxy.



If this body is moving uniformly, then it will keep its velocity constant forever. Can we conclude on figure above, that the body will eventually reach point B, which is placed on the imagined straight line, towards which the body appears to be moving in point A?

The answer is – absolutely not. The body will be subject to acceleration, caused by gravity of the whole cosmos, with dominating acceleration by the host galaxy. On the periphery of Milky Way, acceleration towards the center of galaxy is in the range of 10^{-11} m/s^2. Under even such small acceleration, the body will be traveling in closed elliptical path, and in approx. 250 million years will reach the starting point A again.

This is clearly not uniform motion. As acceleration is continual, at any point in time we can only observe tangential velocity of the body, and tangential velocity is changing continually.

What has this got to do with Lorentz transformations then?

As relativists still want to apply Lorentz transformations in real world, they must then satisfy with approximation that a system appears to be inertial in infinitesimal periods of time, and under small accelerations. But how much acceleration is too much? When does the system stop being inertial – is it when acceleration is in the range of 10^{-100} m/s^2, or 10^{-10} m/s^2, perhaps 1 m/s^2 is also acceptable for ignoring? All experiments were done on Earth under dominant acceleration of about 10 m/s^2. So in such cases, Lorentz transformations apply on real world, right?

Then we must assume that both length contraction and time dilation occur when a body is accelerated, but we must be careful to apply Lorentz transformations in infinitesimal periods of time, and only when acceleration is very (arbitrarily) small.

If time dilation in an inertial frame is calculated with formula:

$$\Delta t' = \frac{\Delta t}{\sqrt{1-\frac{v^2}{c^2}}} \tag{1}$$

where Δ symbol represents any period of time measured in the same place, we must use differential form of (1) for accelerated motion:

$$dt' = \frac{dt}{\sqrt{1-\frac{v^2}{c^2}}} \tag{2}$$

In this equation, speed v is not constant, unless acceleration is centripetal. But if it is centripetal – v is constant, then by integrating we get back to (1). That means that Lorentz transformations in unchanged form are applicable both on uniform and constant circular motion. It does not violate mathematics of Lorentz equations, but it already violates interpretation that they are to be applied in inertial frames only.

In case of linear acceleration, we have:

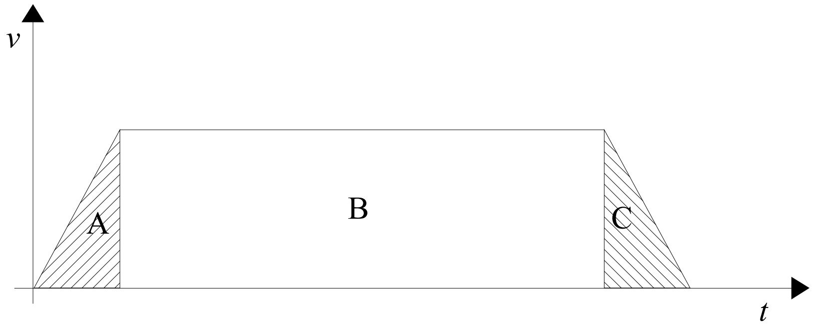
$$v = at \tag{3}$$

$$dt' = \frac{dt}{\sqrt{1-\frac{(at)^2}{c^2}}} \tag{4}$$

By integrating we get:

$$t' = \frac{c}{a} \arcsin \frac{at}{c} \tag{5}$$

Meaning of (5) can be illustrated with the following graph:



A body is set in motion, starting from still position, gaining speed with constant acceleration in segment A until it reaches constant relativistic speed which it maintains throughout segment B, and decreasing speed (negative acceleration) in segment C until the body is at rest.

Equation (5) is applicable on segments A and C, and (1) on segment B.

If we can make arbitrarily strong acceleration in segments A and C, time spent in these segments can be also arbitrarily small. Thus we are left with (1) to calculate period spent on a journey where (almost) all of the time was spent moving at constant speed.

Final conclusion

If we are going to apply Lorentz transformations on any system in reality, then we are always applying them on accelerated motion, as it is the only motion that there is.